80% chance of rain

Let A_j be the event of rain at 9 a.m. on day j this term $0 \leq j \leq n$

Suppose the event A have probability P independently

1. Event and probabilities

Consider an "experiment" which has a set of Ω of outcomes

$$\omega \in \Omega$$

For example

a. tossing a coin $\Omega = \{H | T\}$

Head Tail

b. throwing a dice

$$\Omega \ \{(i,j); \ i,j \in [1,2,3,4,5,6]\}$$

let make them distingwishable  

a red one a blue one

$\{(i,j);\ i,j \in [1,2,3,4,5,6]\}$

A subset of Ω is called an event

For example

a) coin comes up tail $A=\{T\}$

b) We observe of total of 9

$A=\{(3,6),(4,6),(5,4),(6,3)\}$ 

If $\omega \in \Omega$ is the outcome, we say that A occur if $\omega \in A$

Compliment of A:

A^c occurs if A does not occur

Union $A \cup B$ occur if A or B occurs (or both)

Intersection: $A \cap B$ occurs if both A and B occurs

Set difference: $A \setminus B = A \cap B^c$ occur if A occurs and B does not occur

A and B are disjoint if
$$A \cap B = \emptyset$$
i.e. A and B cannot occur together

We assign a probability $P(A)$ of each A

Simplest case:

Ω is finite and all outcomes are equally likely

Then $P(A) = \dfrac{|A|}{|\Omega|}$

Probability of any event A

a) $\left.\begin{array}{l} |\Omega| = 2 \\ |A| = 1 \end{array}\right\} \Rightarrow P(A) = \dfrac{1}{2}$ H T

b) $\left.\begin{array}{l} |\Omega| = 36 \\ |A| = 4 \end{array}\right\} \Rightarrow P(A) = \dfrac{4}{36} = \dfrac{1}{9}$

Elementary combinatorics

Arrangements of distinguishable or different objects

Suppose we have **n** distinguishable objects

? How many ways are there to order them? (permutation)

e.g. $1, 2, 3, 4, \ldots n$

or $n, n-1, n-2, \ldots, 2, 1$

There are n options for ordering the first objects

then $n-1$ — 2nd —

inductively n — $(m-1)$ — mth

until nth

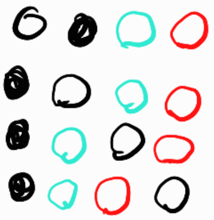
So in all there are

$$n(n-1)\ldots 2\cdot 1 = n! \text{ Permutations}$$

Example

m 1st 2nd 3rd 4th

1st



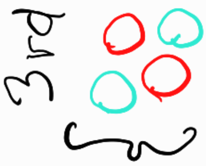
There are n options for the 1st object

or n positions

2nd

once you've chosen the 1st object

$n-1$ remaining options for 2nd object



3rd

2

$n-1$ options for the 2nd

$n-2=2$ $\quad n-2$

4th

So $\quad n -$ 1st

$n-1$ - 2nd

$n-(m-1)$ 3rd

until 1 — last n^{th} 4th

Example

There are $6!$ ways to order the letters of GALOIS

Evariste Galois
(20: 1811 – 1832)
French math

If randomly reorder the letters what is probability that the vowels (A, O, I) are all before consonants? (G, L, S)

"uniformly of random"

There are $3! * 3! = 36$

arrangent of 3 vovels and then 3 consonants

If Ω is the set of all arrangements and $A \subseteq \Omega$ the set of arrangements with all vovels before all consonants, then

$$P(A) = \frac{|A|}{|\Omega|} = \frac{3! \cdot 3!}{6!} = \frac{36}{720} = \frac{1}{20}$$

So you can try the same with other words

Arrangements when not all objects are indistinguishable

How many different arrangemens of
A, A, A, B, C, D

If we had

$A_1\ A_2\ A_3\ B\ C\ D$ there would be $6!$ orderings

$B\ A_2\ D\ A_1\ A_3\ C$
$B\ A\ D\ A\ A\ C$
$B\ A_1\ D\ A_2\ A_3\ C$

correspond to the same ordering of AAA BCD

Each one is one of $3!$ which differ only in the order of $A_1\ A_2\ A_3$

So the $6!$ fall into groups of size $3!$
which are indistinguishable
if $A_1 = A_2 = A_3 = A_4$

We want to count the number of groups which is $\frac{6!}{3!}$

Generalizations

The number of the N objects

$$\underbrace{x_1, \ldots, x_1}_{m_1}, \underbrace{x_2, \ldots, x_2}_{m_2}, \ldots, \underbrace{x_k, \ldots, x_k}_{m_k}$$

where $n = m_1 + m_2 + \ldots + m_k$

$$\frac{n!}{m_1!\, m_2! \cdots m_k!}$$

Case $k = 2$

Ordering of $\underbrace{v, v, \ldots, v}_{m}\ \underbrace{x, x \ldots x}_{n-m}$

is $\dfrac{n!}{m!\,(n-m)!} = \dbinom{n}{m} = \left(\dfrac{n}{n-m}\right)$

Binominal coefficient

Binominal coefficient (which you have written as ${}^{n}C_{m}$)

But here in Oxford we very much like this way of writing $\boxed{\binom{n}{m}}$

That binomial coefficient is something that you are familiar with and it usefull in many way, not just in order to find arrangements of objects, but also to choose M of the given objects from the total N.

And you can do that with ties ✓ and crosses ✗

for example to work out how many are there to form football team of size M from squad of size N just by identifying football team as the players where you assign a tick ✓ whereas the other players are assign as ✗ and so counting

John ✗
Jack ✓
Mike ✓
⋮
Tom ✗
Frank ✓

how many ways there are to
select team is exactly the same
as number of ways to order
the order the tics $\boxed{\checkmark}$ and crosses ✗
but I haven't got time to
write that

I leave you with that to maybe
think about more, if you
haven't put this together
at this stage

So this leave us to say :
what about

" 80% chance of rain on Friday "

I will see you later on next lection... anyway

Oxford