Arrangements when not all objects are indistinguishable

? How many different arrangemens of

$\frac{6!}{3!} = 120$ $\underbrace{A, A, A}_{A \text{ is repeated}}, \underbrace{B, C, D}_{3 \text{ are distinguishable}}$

If we had

$A_1\ A_2\ A_3\ B\ C\ D$ there would be 6! ordering

$B\ A_2\ D\ A_1\ A_3\ C$
$B\ A\ D\ A\ A\ C$
$B\ A_1\ D\ A_2\ A_3\ C$

correspond to the same ordering of A A A B C D

Each one is one of 3! which differ only in the order of $A_1\ A_2\ A_3$

So the 6! fall into groups of size 3!
which are indistinguishable

3rd

n − 1 options for the 2nd

$n-2=2$ $n-2$

2

4th

So

$n -$ 1st

$n - 1$ − 2nd

$n-(m-1)$ 3rd

until 1 — last n^{th} 4th

Example

There are 6! ways to order the letters of GALOIS

Evariste Galois (20: 1811 − 1832).

French math

If randomly reorder the letters what is probability that the vowels (A, O, I) are all before consonants? (G, L, S)

"uniformly of random"

There are $3! * 3! = 36$

$$P(A) = \frac{3}{6} \cdot \frac{3}{5} \cdot \frac{2}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} \cdot 1 = \frac{36}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} = \frac{6}{5 \cdot 4 \cdot 3 \cdot 2} = \frac{1}{20}$$

arrangent of 3 vovels and then 3 consonants

If Ω is the set of all arrangements and $A \subseteq \Omega$ the set of arrangemen with all vovels before all consonont then

$$P(A) = \frac{|A|}{|\Omega|} = \frac{3! \cdot 3!}{6!} = \frac{36}{720} = \frac{1}{20}$$

So you can try the some with other words

80% chance of rain

Let A_j be the event of rain at 9 a.m. on day j this term

30 - 40 days

$0 \leq j \leq n$

Suppose the event $\underline{A}$ $\underline{\text{have}}$ probabili: independently P

0.1 ... 0.9

1. Event and probabilities

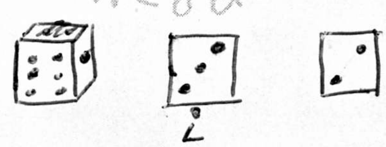
Consider an "experiment" which has a set of Ω of outcomes

$$\omega \in \Omega \leftarrow \text{sample space}$$

For example

a. tossing a coin $\Omega = \{H | T\}$

Head Tail

b. throwing a dice 

$$\Omega \{(i, j); i, j \in [1, 2, 3, 4, 5, 6]$$

set of possible outcomes

let make them distingwishable 

That binomical coefficient is something that you are familar with and it usefull in many way, not just in order to find arrangement of objects, but also to choose M of the given objects from the total N.

And you can do that with ties V and crosses X

for example to work out how many are there to form football team of size M from squad of size N just by identifying football team as the players where you assign a tick V whereas the other players are assign as X and so counting

John X
Jack V
Mike ✓
⋮
Tom X

$$\frac{n!}{m_1!\, m_2! \cdots m_k!}$$

Case $k = 2$

Ordering of $\underbrace{v, v, \ldots, v}_{m}$ $\underbrace{x, x \ldots x}_{n-m}$

is $\dfrac{n!}{m!\,(n-m)!} = \binom{n}{m} = \binom{n}{n-m}$

$\uparrow$

Binominal coefficient

Binominal coefficient (which you have written as ${}^{n}C_{m}$)

But here in Oxford we very much like this way of writing $\boxed{\binom{n}{m}}$

how many ways there are to
select team is exactly the same
as number of ways to order
the order the tics $\boxed{\checkmark}$ and crosses
✗
but I haven't got time to
write that
I leave you with that to maybe
think about more, if you
haven't put this together
at this stage

We want to count the number of groups which is $\frac{6!}{3!}$

Generalizations

The number of the N objects

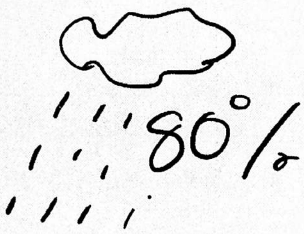
$$\underbrace{x_1, \ldots x_1}_{m_1}, \underbrace{x_2, \ldots x_2}_{m_2}, \ldots, \underbrace{x_k \ldots x_k}_{m_k}$$

where $n = m_1 + m_2 + \ldots + m_k$

So this leave us to say
what about

"80% chance of rain on
Friday"

I will see you later
on next lection... anyway



Oxford

A and B are <u>disjoint</u> if

$$A \cap B = \emptyset$$

i.e. A and B cannot occur together

We assign a probability $P(A)$ of each A

Simplest case:

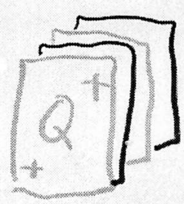
Ω is finite and all outcomes are equally likely

Then $P(A) = \dfrac{|A|}{|\Omega|}$

Probability of any event A

a) H T $\left.\begin{array}{l}|\Omega| = 2\\ |A| = 1\end{array}\right\} \Rightarrow P(A) = \dfrac{1}{2}$ H T

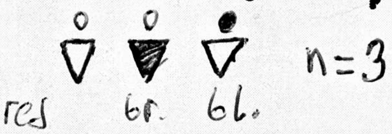
b) $\left.\begin{array}{l}|\Omega| = 36\\ |A| = 4\end{array}\right\} \Rightarrow P(A) = \dfrac{4}{36} = \dfrac{1}{9}$



<u>Elementary combinatorics</u>

Arrangements of distinguishable or different objects

Suppose we have **n** distinguishable objects



How many ways are there to order them (permutation)

$\Omega = 3! = 3 \cdot 2 \cdot 1 = 6$

e.g. $1, 2, 3, 4, \ldots n$

or $n, n-1, n-2, \ldots 2, 1$

There are n options for ordering the first objects

then $n-1$ — 2nd —

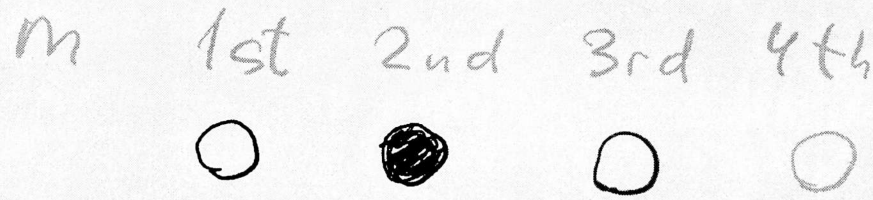
inductively n — $(m-1)$ — m^{th}

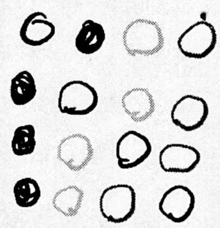
until n^{th}

So in all there are

$n(n-1) \ldots 2 \cdot 1 = n!$ Permutations

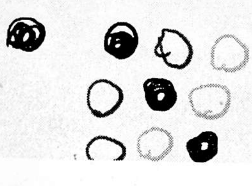
<u>Example</u>





There are n options for the 1st object or n positions

Once you've chosen the 1st object



$n-1$ remaining options

$\{(i,j), i,j \in [1,2,3,4,5,6]\}$

A subset of Ω is called an event

For example

a) coin comes up tail $A = \{T\}$

b) We observe of total of 9

$A = \{(3,6), (4,5), (5,4), (6,3)\}$ 

If $\omega \in \Omega$ is the outcome, we say that A occur if $\omega \in A$

Compliment of A:

A^c occurs if A does not occur

Union $A \cup B$ occur if A or B occurs (or both)

Intersection: $A \cap B$ occurs if both A and B occurs

Set difference: $A \setminus B = A \cap B^c$ occur if A occurs and B does not occur